Paper Reference 9MA0/01
Pearson Edexcel
Level 3 GCE

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Wednesday 7 October 2020 – Afternoon

Time: 2 hours plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green), calculator

ITEMS INCLUDED WITH QUESTION PAPER
Diagram Book
Answer Book

Y66785A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams − there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **16** questions in this Question Paper.
The total mark for this paper is **100**

The marks for each question are shown in brackets − use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Turn over

1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3 marks)

(b) Explain how you could use $x = \dfrac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2 marks)

(Total for Question 1 is 5 marks)

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2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(Total for Question 2 is 3 marks)

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3. Relative to a fixed origin O

- point A has position vector $2\underline{i} + 5\underline{j} - 6\underline{k}$
- point B has position vector $3\underline{i} - 3\underline{j} - 4\underline{k}$
- point C has position vector $2\underline{i} - 16\underline{j} + 4\underline{k}$

(a) Find $\overrightarrow{AB}$

(2 marks)

(continued on the next page)

3. continued.

(b) Show that quadrilateral OABC is a trapezium, giving reasons for your answer.

(2 marks)

(Total for Question 3 is 4 marks)

Turn over

4. The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2 marks)

(b) Show that

$$ff(x) = \frac{ax+b}{x-3}$$

where a and b are integers to be found.

(3 marks)

(Total for Question 4 is 5 marks)

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5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an arithmetic sequence,

(a) find the fastest speed of the car in 3rd gear.
(3 marks)

(continued on the next page)

5. continued.

Given that the fastest speed of the car in successive gears is modelled by a geometric sequence,

(b) find the fastest speed of the car in 5th gear.

(3 marks)

(Total for Question 5 is 6 marks)

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6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.
(3 marks)

(continued on the next page)

6. continued.

The temperature, $\theta°C$, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right)$$

$$0 \leq t < 24$$

where t is the number of hours after midnight.

(continued on the next page)

Turn over

6. continued.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1 mark)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3 marks)

(Total for Question 6 is 7 marks)

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7. Refer to the diagram for Question 7 in the Diagram Book.

It shows a sketch of a curve C with equation $y = f(x)$ and a straight line L

The curve C meets L at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and L as shown in the diagram.

(continued on the next page)

7. continued.

Given that

- **$f(x)$ is a quadratic function in x**
- **$(-2, 13)$ is the minimum turning point of $y = f(x)$**

use inequalities to define R

(Total for Question 7 is 5 marks)

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8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

(continued on the next page)

8. continued.

Use this information to write down a suitable equation for n in terms of t

(You do not need to evaluate any unknown constants in your equation.)

(Total for Question 8 is 2 marks)

9. Refer to the diagram for Question 9 in the Diagram Book.

It shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = 8(2 + x - x^2)e^{-2x}$$

(3 marks)

(continued on the next page)

9. continued.

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C

(3 marks)

(continued on the next page)

9. continued.

The function g and the function h are defined by

$$g(x) = 2f(x) \qquad x \in \mathbb{R}$$
$$h(x) = 2f(x) - 3 \qquad x \geq 0$$

(c) Find

(i) the range of g

(ii) the range of h

(3 marks)

(Total for Question 9 is 9 marks)

Turn over

10. (a) Use the substitution $x = u^2 + 1$

to show that

$$\int_5^{10} \frac{3\,dx}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,du}{u(3+2u)}$$

where p and q are positive constants to be found.

(4 marks)

(continued on the next page)

10. continued.

(b) Hence, using algebraic integration, show that

$$\int_{5}^{10} \frac{3\,dx}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6 marks)

(Total for Question 10 is 10 marks)

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11. Refer to the diagram for Question 11 in the Diagram Book.

Circle C_1 has equation

$$x^2 + y^2 = 100$$

Circle C_2 has equation

$$(x - 15)^2 + y^2 = 40$$

The circles meet at points A and B as shown in the diagram.

(continued on the next page)

11. continued.

(a) Show that angle $AOB = 0\cdot635$ radians to 3 significant figures, where O is the origin.

(4 marks)

The region shown shaded in the diagram is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4 marks)

(Total for Question 11 is 8 marks)

12. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$$
$$\theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3 marks)

(continued on the next page)

12. continued.

(b) Hence, or otherwise, solve for

$0 < x < 180°$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$$

(5 marks)

(Total for Question 12 is 8 marks)

Turn over

13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3 marks)

(continued on the next page)

Turn over

13. continued.

Remember:

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

(b) For this sequence explain why $k \neq 1$

(1 mark)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3 marks)

(Total for Question 13 is 7 marks)

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14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3 marks)

(continued on the next page)

14. continued.

Given that

- **the initial radius of the balloon is 40 cm**
- **after 5 seconds the radius of the balloon is 20 cm**
- **the volume of the balloon continues to decrease at a constant rate until the balloon is empty**

(b) solve the differential equation to find a complete equation linking r and t
(5 marks)

(continued on the next page)

14. continued.

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2 marks)

(Total for Question 14 is 10 marks)

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4 marks)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3 marks)

(Total for Question 15 is 7 marks)

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16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(Total for Question 16 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
